補題 $\frac{\partial(F,G)}{\partial(x,y)} = \frac{\partial(F,G)}{\partial(f,g)}\frac{\partial(f,g)}{\partial(x,y)}$ — ①

②: 補題を用いて系の3, の4 を示せ

系の3 $\left(\frac{\partial z}{\partial x}\right)_y = \left(\frac{\partial z}{\partial f}\right)_y \left(\frac{\partial f}{\partial x}\right)_y$

① で $F=z$ $g=G=y$ とおくと 左辺 $= \frac{\partial(z,y)}{\partial(x,y)} = \left(\frac{\partial z}{\partial x}\right)_y$

右辺 $= \frac{\partial(z,y)}{\partial(f,y)}\frac{\partial(f,y)}{\partial(x,y)} = \left(\frac{\partial z}{\partial f}\right)_y \left(\frac{\partial f}{\partial x}\right)_y$ となり 示せた

系の4 $\left(\frac{\partial f}{\partial x}\right)_y \left(\frac{\partial x}{\partial f}\right)_y = 1$

① で $F=x$ $g=G=y$ とおくと 左辺 $= \frac{\partial(x,y)}{\partial(x,y)} = 1$

右辺 $= \frac{\partial(x,y)}{\partial(f,y)}\frac{\partial(f,y)}{\partial(x,y)} = \left(\frac{\partial x}{\partial f}\right)_y \left(\frac{\partial f}{\partial x}\right)_y$ となり 示せた